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A model is developed which relates an index of school performance to management policy under a variety of environmental conditions. The process of education is defined by the qualitative concept that education is a process of communication. In the model development, the assumed relationships among the variables (factors controllable by management and factors determined largely by the nonschool environment) are first listed and then a mathematical analog to the statements is developed. At the present stage of development the model relates changes in achievement test scores (on a school by school basis) to pertinent resource input factors. Data from a sample of schools is used to estimate parameters and to test the model's ability to predict performance from management policy. The results of the empirical work are presented and practical implications for further research and for school system administrators are discussed. (HW)

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ON THE MODELING OF RELATIONSHIPS BETWEEN PERFORMANCE AND RESOURCE MANAGEMENT IN AN URBAN SCHOOL DISTRICT

by

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ABSTRACT

Top management in many urban public school systems are calling upon systems analysts and operations researchers to help in making their schools more effective. Recently considerable effort has been devoted to improving the planning and resource allocation functions in urban school systems. A critical research problem in this area is the development of models which relate an acceptable index of school performance to management policy under a variety of environmental conditions.

This paper reports one approach to the problem. The process of education is defined by a qualitative concept. The key concept is that education is a process of communication. The definitions include factors controllable by management and factors determined largely by the non-school environment. In the model development, the assumed relationships among these factors (variables) are first listed and then a mathematical analog to the statements is developed.

At the present stage of development the model relates changes in achievement test scores (on a school by school basis) to pertinent resource input factors. Data from a sample of schools has been used to estimate parameters and to test the model's ability to predict performance from management policy. The results of the empirical work are presented. Practical implications both for further research and for school system administrators are also discussed.

I. INTRODUCTION

Administrators of large city school districts have serious problems. New demands have been placed on city schools as a result of shifting population and increasing technological complexity now emphasized by rapid unionization of teachers and by civil rights organizations. Prior to these developments there were few crises which city school administrators could not handle within the resources made available by the community. Now when more resources, both money and trained personnel, are desparately needed they are getting harder to obtain. There are a number of reasons for this, particularly the changing nature of the tax structure and tax base in cities. Under these conditions it is important that school systems operate efficiently. For whatever underlying causes, school district administrators have begun to take on active interest in management science. They have been turning to systems analysis and operations research in hope of solving some of the problems.

Operations researchers have done considerable work in a variety of problem areas of interest to school district management. Recently a large conference in Washington centered upon "Operational Analysis of Education". There were reports on the use of simulation models in educational financial planning, mathematical programming applications in selecting attendance boundaries for schools and many

other applications.[1, 2, 3] But with all of this, operations researchers have not had a significant impact on one of the most critical problems, those involving the teaching-learning processes. For example, little work has been done on relating uses of resources in the educational system to a measure of <u>educational</u> output.

For example, if students are to be bussed around the city, it should be done at minimum cost. The problem is the same as distributing goods in a company at least cost. But with marketable products the distribution expense has a clearly defined relationship to sales and ultimately to profits. Once we have transported students around a city at least cost, what is our profit? How has education been improved?

The same type of question can be asked about the effects of building more schools, hiring more teachers, changing curriculum, buying books or installing computer based instruction systems. So far there has been little work done on developing explicit relationships between the amount and mix of resources and some measure of system performance other than input.

We have been working for administrations of an urban school district on improving their planning processes. The effort has resulted in two simulation models. The first is very aggregate and

produces forecasted budgets given certain policy inputs. The second simulator is a more detailed representation and is centered around educational programs [4 and 5]. The first aggregate model, called School District One did not relate resource allocations or policies to an index of educational performance. S.D. One is only capable of forecasting financial and operational consequences of district wide management policy. For example, it is useful in analyzing the financial impact of shifts in population or enrollment or the effects of union contract settlements with teachers' unions. This model's inability to estimate the impact of various policies on education is a severe limitation, of course. This limitation provided the main motivation for our second model, S.D. Two. This model with its educational program orientation is designed to include an educational achievement forecasting sub-model. This sub-model relates a measure of change of student achievement to educational program resources used with the students. This paper reports on our first approach to the development of this achievement forecasting submodel.

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II. THE SYSTEM UNDER STUDY

The subject of study is an urban school district consisting of primary and secondary schools. A review of the literature in the

field of educational administration indicates that there is no simple framework for evaluating the performance of schools or of school districts. So many people or groups are involved in school evaluation that a wide variety of indices and measures are used depending upon the circumstances and the questions being asked. This might be so either because urban schools are involved in a number of activities each furthering a different set of purposes. On it may be that the schools have one main purpose, "to educate" and we are unable to define and measure it. In either case, if management (the superintendent of schools or board of education) is to exercise control over the system, they must be able to gauge system performances. They also must be able to relate the effect of alternative policies on this guage.

In our work we have by-passed some of the questions about goals and objectives of the school system. We use a concept of the educational process that allows the development of the performance forecasting tool; a model based upon stated assumptions about the education process.

III. A DEFINITIONAL SYSTEM: EDUCATION AS A COMMUNICATION PROCESS

The basic unifying concept is to view education as a communication process, in particular between adults and children. This

view simplifies the organization of relevant factors and hypotheses. It leads to a series of statements about education. The modeling process consists of developing a mathematical or symbolic analog to these statements.

At a gross level, school systems engage in a form of communication to change the future or potential behavior of students. It is also important to note that with children others, particularly parents, engage in communication with similar effects. The communication process goes on over a time period. In modeling this time period must be specified explicitly. Other elements in the process are:

1) The individuals.

   - Receivers -- generally students.
   - Senders or managers of the communication sending process.

     --teachers [and other students, but we will include other students as an environmental factor as a simplifying approach].

     --parents

     --other members of the students' communication.

2) Present and past environments of the individuals involved

   - Past educational experiences.
   - Physical environment.
   - Other students, peers and siblings.

3) Instruments of communication.

4) The messages to be communicated.

Some of these factors can be controlled. The educational process is managed by identifying and changing these controllable elements. To illustrate this we will consider some examples, starting with the "senders".

Even though students in a given school communicate with one another we shall assume that the prime "senders" are the teachers and treat inter-student communication as environmental. The teachers can be physically reassigned or given new functions. Management often reassigns teachers to classes of students. Functionally, teachers can be changed by controlling the roles and duties which they are to perform by curriculum changes or through staff development programs. It is possible that a reassignment of teachers or a change in the number of students per teacher, can bring about a functional change. Care must be taken in measuring these changes.

In one summer program that we know of, teachers were put with classes of six students each. The program lasted for six weeks. It was found that for many of the teachers there was a period of as long as five weeks when they continued to teach this small class of

six students in the same way that they had treated groups of thirty or more. It is clearly necessary to identify precisely how the changes in controllables will influence the communication process.

Most public school systems have little control over the receivers in the communication. The students must be accepted. Attention to student health is one effort to insure reception is facilitated.

Alterations in the students' environments are another class of changes. The locations of schools can be changed, so can their size, shape or design. Bussing or other measures to change racial mix change the in-school environment. The individuals can be transported to totally different surroundings as in summer camp programs. There are wide variety of competitive and reward devices which can influence receiver-receiver patterns or sender-receiver patterns.

One of the major environmental factors is the parent-student communication process. In fact, it is so important that a school district may have to make an effort to influence it. If it does, the expenditures for influencing parental participation become a controllable variable; an important use of resources.

Turning to the other elements in the communication process, we may vary the instruments used in the communication process. The teacher's chalk and blackboard would be instruments in this sense.

Films or slides could replace blackboard and chalk. The teacher can 'send' not only material he originates but material created by others. Thus, the teacher managers the use of texts and instruments for presenting material. It is possible to make the instrument self-managing to some extent as in computer-assisted instruction.

Finally, there is a wide range of possible changes in the messages which are used in the communication process. The content of the message may be changed by replacing one subject with another. The structure of messages can be changed by changing the order in which they are presented. Messages may be designed to perform various functions: motivational, disciplinary or instructional.

All of these factors (and others influenced by management policy) must be related to system output. The method used here to relate some of the important controllable elements of the communication process to output is (1) to state the assumed and observed relationships, (2) to develop a mathematical analog to those relationships, (3) to validate the resulting model. In the next section we formulate the basic statements. It is worth repeating that, initially, we have treated only some of the most basic aspects of the whole process.

IV. PERFORMANCE MODEL

The model is designed to gauge educational activity in the aggregate. Since the educational activity goes on continuously it is

necessary to refer to a particular unit of time (such as a year). A reference group of students must be defined. The grouping of students assumed in constructing the model depends upon the use which is to be made of the model itself. In view of the way most schools operate a good unit of aggregation would be the individual classroom. For practical reasons (data availability), so far we have had to use elementary schools as the unit of aggregation. The index of education, which is the dependent variable of this model, is the change in median achievement test scores over the period of interest.

The performance model development starts with the following statements about the education process. These are generally drawn from the statements of educators and administrators.

(1) The change in student achievement over time will depend upon formal educational efforts in the school and upon less formal education in the home.

(2) The parents of students are the main producers of the home or non-formal contribution to overall achievement.

(3) The non-school education is dependent upon the level of parental education.

(4) Within the school the more teaching-staff time devoted to the students, the greater the expected change in achievement, but with eventual diminishing returns. If the number of pupils per teacher

is small (e.g., 5 to 1), the added benefit of more teaching time declines. Paraprofessionals (non-certified teaching staff) are treated as though their time is the same as teachers' time.

(5) The contribution to in-school education made by spending money on educational materials (e.g., books) is proportional to the amount spent.

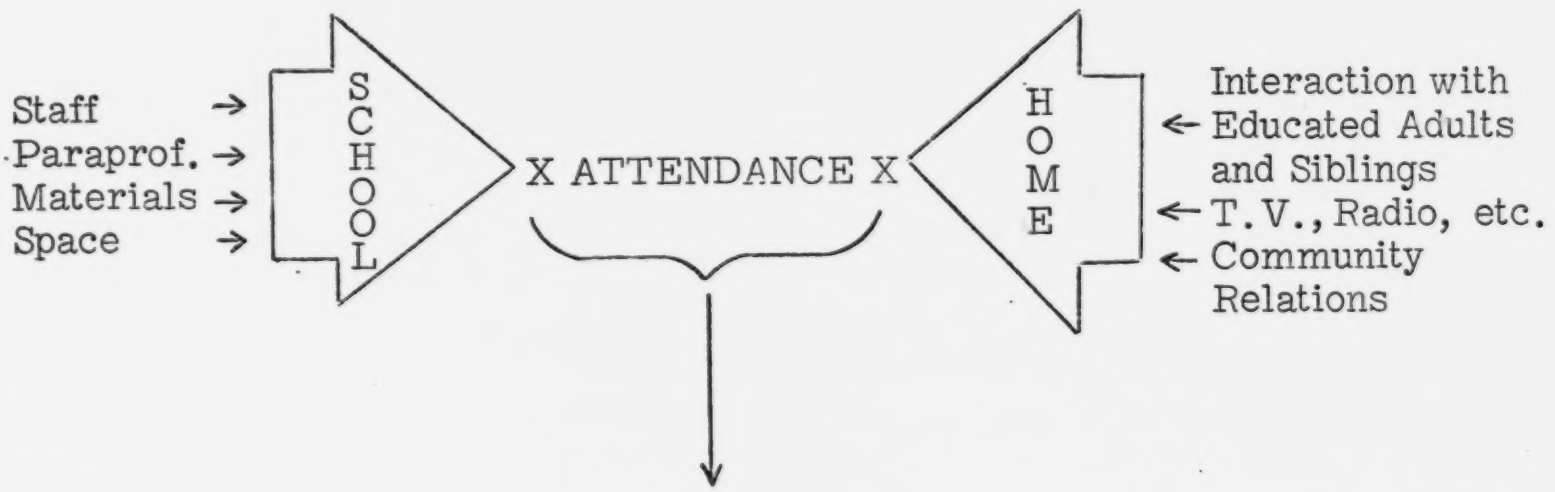
(6) Given the level of expenditure on staff, books, and para-professionals in a school, the less crowded that school is, the larger the benefit to be expected in education. As the school becomes less and less crowded the successive additions of space per student produce smaller added benefits.

(7) The in-school and non-school education interest when the students attend school. The extent (not content) of this interaction is indicated by the percentage attendance at school over the time period of interest. (It is recognized that attendance can also be considered a result of the educational process.)

(8) The results of the education effort during a period of time is approximately indicated by the change in median scores on Iowa Tests of Basic Skills. The data available for this study make it necessary to use scores covering two grades, from 4th to 6th. The data was cross sectional, suitable longitudinal data will be gathered in future tests.

Earlier we said that the school district administration might have to influence the communication which takes place in the non-school environment. The model does not yet include expenditures for influencing parents (community-relations expenditures) as no data is available to test such a model. The form of this influence has been hypothesized.[1]

The diagram below illustrates schematically how these factors are related in the performance model.



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[1] Community relations expenditures are designed to modify the quality or amount of non-school communication. The effect of these expenditures in changing achievement increases as the quality of the non-school environment (indicated by parental and peer group education) declines.

According to this structure we would expect a low level of performance if a). The schools devoted insufficient resources to providing teachers materials and space b). If the students' home backgrounds were relatively bad (in terms of their parents' education level) and c). If the students do not attend the school in which they are enrolled. Lower performance might be caused by some combination of these as well.

This model is stated below symbolically.

$$\text{Change in Test Score} = \Big[f(\text{Staff} + \text{Paraprof.}) + f(\text{Mat'l}) \Big] \times f(\text{Area}) \times \text{Attend.} \times \Big[f(\text{Socio-Econ}) \Big]$$

We have developed several variations on this basic model. These differ in the mathematical functions which are used to express relationships among variables. Several of the factors (teaching staff and parental educational level) are hypothesized to enter into the education process with diminishing returns. These factors are represented mathematically by logistic functions of the form:

$$f(s) = \frac{k}{1 + e^{a + bx}} \qquad k,\ a,\ b \text{ are all constants}$$

These functions were selected because they correspond with the qualitative statements of the relationships discussed by educators.

The influence of area per student, A, is represented in the index by a simple exponential curve of the form:

$f(A) = k(1-e^{A/c})$ where k, c are both constants.

The material contribution is linear. This version of the model and the definitions of its variables appears as follows:

$$I_i = \left[\frac{c_1}{1+e^{c_2+c_3(s_i+P_i)}} + c_4 M_i\right] c_s (1-e^{-A_i/c_0}) \alpha_i \left[\frac{c_7}{1+e^{c_8+c_9 L_i}}\right] \quad (1^*)$$

where:

I = Index value (change in achievement test score over the reference period).

S = Staff per student.

P = Paraprofessionals per student.

M = Materials per student (in $/student)

A = Area per student (sq. ft./student)

α_i = Attendance in school i

L = Measure of socio-economic background (parental education)

i = Subscripting variable devoting individual school (i = 1, 2, ... N)

$c_0 \ldots c_9$ are all constants to be estimated from data.

As the model (1) has been written it is obviously static. We have not explicitly included any of the relationships which might exist between

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*If community relations expenditures were included, we would modify the right hand bracket by adding a term $g(L_i) \cdot f(x_i)$ where x_i is representative of the level of expenditure per student. The term would become: $\frac{c_7}{1+e^{c_8+c_9 L_i}} + q(L_i) \quad f(x_i)$

the change in achievement during the reference period and prior periods. To some extent the attendance term may serve this purpose implicitly. Attendance itself may be a function of past educational experience, as well as a function of the level of enforcement of compulsory attendance laws.

Again, we are dealing with elementary schools in this case and so we are not making distinctions between the types of teachers or achievement in particular skills. At the fourth grade the activities within the school are much less highly differentiated than they would be in high school. The dropout rate also less of a problem generally. Ideally we should track the same reference group of students, however, transfers and changes in student population are inevitable.

V. STATISTICAL PROCEDURES

The form of the model (1) makes the job of estimating its 10 parameters more difficult than if a simple linear regression model were used. The normal equations for (1) resulting from a standard squared error minimization are too complex to solve in closed form. We estimated the parameters by a direct numerical approach. We used a search package to minimize a goodness of fit criterion by direct search on parameter values.

Let Y_i be the observed change in median achievement for school i during a period of interest. For each school the value of S, P, M, A, α and L are observed. A value of school i's achievement index may then be computed for a given set of parameter values $c_0, \ldots, c_9$ in the model. We define the error criterion to be the mean squared difference between the actual Y_i and the computed $I_i(c_0 \ldots c_9)$. The average being taken across schools. The parameters estimates being sought are $(\tilde{c}_0, \tilde{c}_1 \ldots \tilde{c}_9)$ which

$$\underset{\underset{\sim}{c}}{\text{Min}} \sum_{i=1}^{N} (I_i(\underset{\sim}{c}) - Y_i)^2. \qquad (2)$$

This estimation procedure is not completely satisfactory. The minimization in (2) has many local minima. The search had to be started from many different points in the space of parameter estimates. The procedure is also quite expensive in terms of computer time if N, the number of data points, is large. We have tried a variation on this model by reparameterization which greatly facilitates estimation. This is discussed below.

In data which has been made available, so far, there has not been much school to school variation in many of the factors. That is the space, teaching time per student, etc. are all relatively uniform. We decided that it would be worth while to examine a version of the

model which would be an approximation to the original model (1) over the region of values on which we had data. To do this we linearized all non-linear terms to get $I_i = [c_1' + c_2' (S_i + c_3' P_i) + c_4' M_i] \frac{A_i}{c_5'} \cdot \alpha_i \cdot (c_c' + c_7' L_i)$ (3) Expanding this, we get the polynomial expression

$$I_i = k_1 A_i \alpha_i + k_2 A_i \alpha_i L_i + k_3 S_i A_i \alpha_i + k_4 S_i A_i \alpha_i L_i$$

$$+ k_5 P_i A_i \alpha_i + k_6 P_i A_i \alpha_i L_i + k_7 M_i A_i \alpha_i + k_8 M_i A_i \alpha_i L_i \qquad (4)$$

where the $k_1 \ldots k_8$ are all products of the $c_1' \ldots c_7'$ from expression (3). We have expressed a model in 7 parameters (3) as a new one in 8 parameters. If we can estimate the k's of expression (4) we cannot go back and compute unique estimates of the c's in (3). But with expression (4) we have a function which is linear in terms of variables defined as products of the original factors. We can estimate parameters of (4) by straightforward regression, after eliminating multicolinearity.

The only standard against which we can hold these models is a simple linear regression on the same variables. The criterion for comparison is the residual variance of each model. We have run the models on data for 28 elementary schools. The table below summarizes the results:

Comparisons of Residual Errors of Models

Model	Parameter d.f.	Error d.f.	Error m.s.	R^2 (S_y^2 = 0.091)
Linear Regression	6	22	0.064	0.296
Expression (4)	8	20	0.046	0.495
Expression (3)	9	19	0.0404	0.556

It is not possible to conclude that either of the two performance models is significantly better than the linear regression at an acceptable α level. (We used the Fisher z statistic to test the last two R^2's against the first.) There are two few observations, however, to draw any final conclusions. We are now attempting to gather more data, and this brings up the final point which we would like to make.

VI. IMPLICATIONS FOR MANAGEMENT AND FOR FURTHER RESEARCH

It is very difficult to gather good data for the type of models building we are attempting. No matter how cooperative the management supporting the effort, if data on some of the variables in the model has never been collected, there is little foundation for rapid progress. Generally, there is data on achievement test results but almost nothing accessible on the process and resource use which is supposed to be producing the results. One implication which can be drawn is that

present analysts and researchers should begin to lay the groundwork for better future research. One way to do this is to begin to assist managers in defining their problems quantitatively and to identify for them the data requirements.

It is interesting to note that in order to help educational managers, we ourselves are going to have to assume the roles of educators as well as researchers.

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